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# Marine Physical Laboratory

## The Interaction of Nonlinear Internal Waves with Coastal Topography and River Outflows

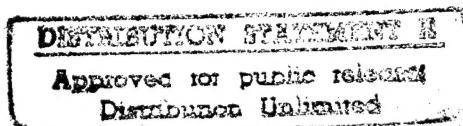
**W. Kendall Melville**

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### **Final Report**

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# The Interaction of Nonlinear Internal Waves with Coastal Topography and River Outflows

**W. K. Melville**

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## *Abstract*

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In this project we conducted analytical and numerical models of the interaction of nonlinear internal waves with coastal topography, and considered models of the evolution of river outflows. Analytical and numerical models of the evolution of nonlinear Kelvin waves showed that they could evolve to breaking along a front for a distance offshore comparable to the Rossby radius. It was found that in rotating systems the time to breaking was delayed when compared to the corresponding non-rotating case. The problem of the propagation of fronts and hydraulic jumps along boundaries in rotating fluids was formulated and solved with an approximate analytical solution and more complete numerical solutions. It was found that asymptotically the front tends to a wave of permanent form near the coast, with an incidence angle offshore which is a function of the amplitude of the front.

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## *Research Summary*

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Internal Kelvin waves play a significant role in the dynamics of the coastal oceans. In recent years a considerable amount of work has been done in both experimental and theoretical research on the nonlinear aspects of the Kelvin wave evolution (see Melville et al. 1989,1990,

Grimshaw and Melville 1989; Tomasson and Melville, 1992). In this research we continue to study those characteristics of internal wave propagation, which are related to wave breaking and thereby to momentum and energy exchange in coastal oceans and the atmosphere.

In the first part of the research (Fedorov and Melville 1993, 1994a), we consider the evolution of nonlinear Kelvin waves using analytical and numerical methods. In the absence of dispersive (nonhydrostatic) effects, such waves may evolve to breaking. In describing such waves, we employ a set of coupled evolution equation for the isopycnal displacement and across-shelf velocity, derived by Melville et al., (1989) and simplified by neglecting dispersion.

We find that one of the effects of rotation is to delay the onset of breaking in time by up to 60%, with respect to a comparable wave in the absence of rotation. This delay is consistent with qualitative conclusions based on transverse averaging of the evolution equations. The onset of breaking occurs almost simultaneously over a zone of uniform phase that is normal to the boundary, and extends over a distance comparable to the Rossby radius of deformation (Fig.1). In other words, the process of breaking embraces the most energetic area of the wave.

In contrast to the linear Kelvin wave, the nonlinear wave develops a dipole structure in the cross-shelf velocity, with a zero net offshore flow (Fig.2). With increasing nonlinearity the flow produces a stronger offshore jet ahead of the wave crest. The Kelvin wave amplitude at the coast decays slightly with time. This and other major features of the wave are accounted for by an analytical model, based on slowly-varying averaged variables and Lagrangian formulation. As part of the analysis it is demonstrated that the evolution of the wave phase may be described by an inhomogeneous Klein-Gordon equation.

Motivated by the study of breaking Kelvin waves and the preliminary finding by Pratt (1983,1987), further we consider three-dimensional hydraulic jumps (shocks) propagating along boundaries in rotating fluids. We refer to them as Kelvin jumps (Fedorov and Melville 1994b). In the lee of the Kelvin jumps, the wave field decays exponentially offshore in a manner similar to that of a Kelvin wave.

We obtain the jump relations and derive an evolution equation for the jump as it propagates along the boundary. It is shown that after some initial adjustment the Kelvin-type jump assumes a permanent form and propagates with a constant velocity along the coast (Fig.3 - Fig.4). At

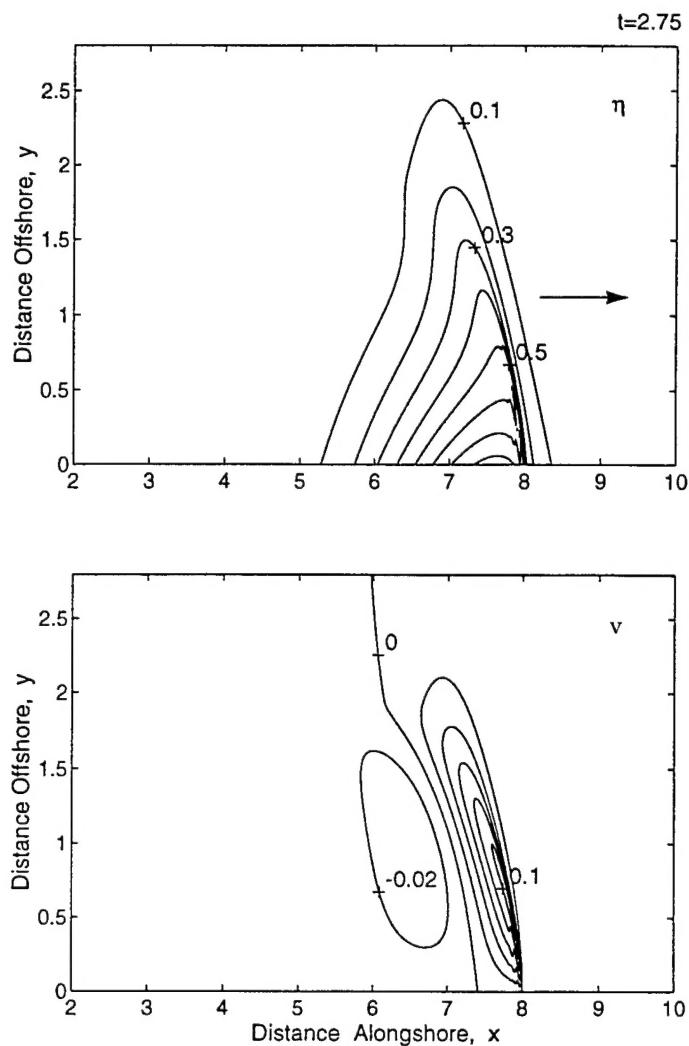


Fig.1 Direct numerical solution giving contour map of the elevation of the interface  $\eta$  and transverse velocity  $v$  before the breaking, which will occur at  $t=3.0$ . The concentrated isolines indicate the region of eminent breaking.

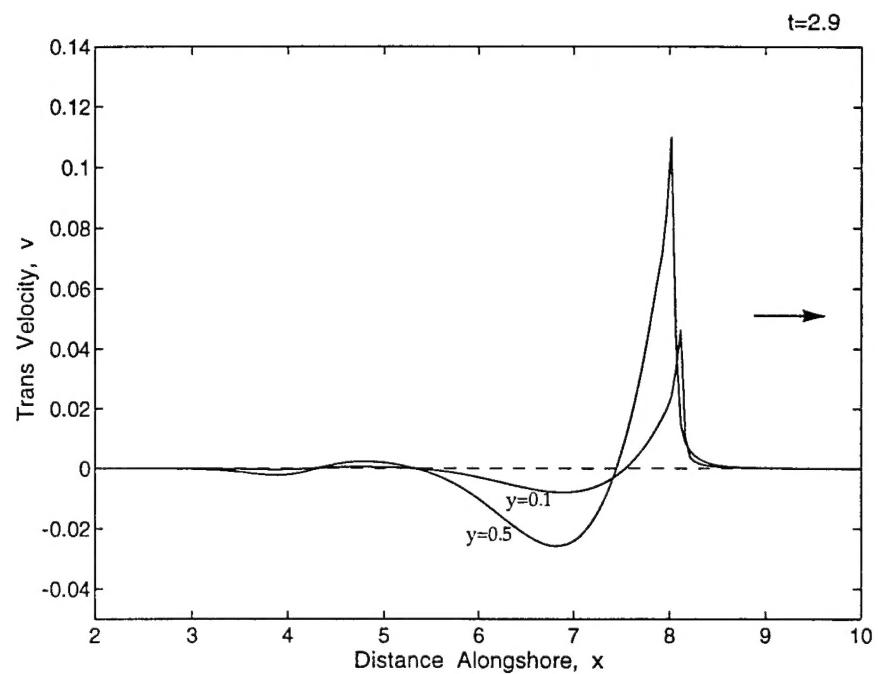


Fig.2 The profiles of the transverse velocity at the time close to breaking, for two distances offshore ( $y=0.1$ ,  $y=0.5$ ). The wave is moving to the right. Positive values of the velocity corresponds to offshore flow, negative to onshore flow. The sharp peak (jet-like flow) is caused by the increasing steepness of the wave. The values of transverse velocity are given in nondimensional variables, with the maximum value corresponding to  $\sim 4\text{cm/sec}$ .

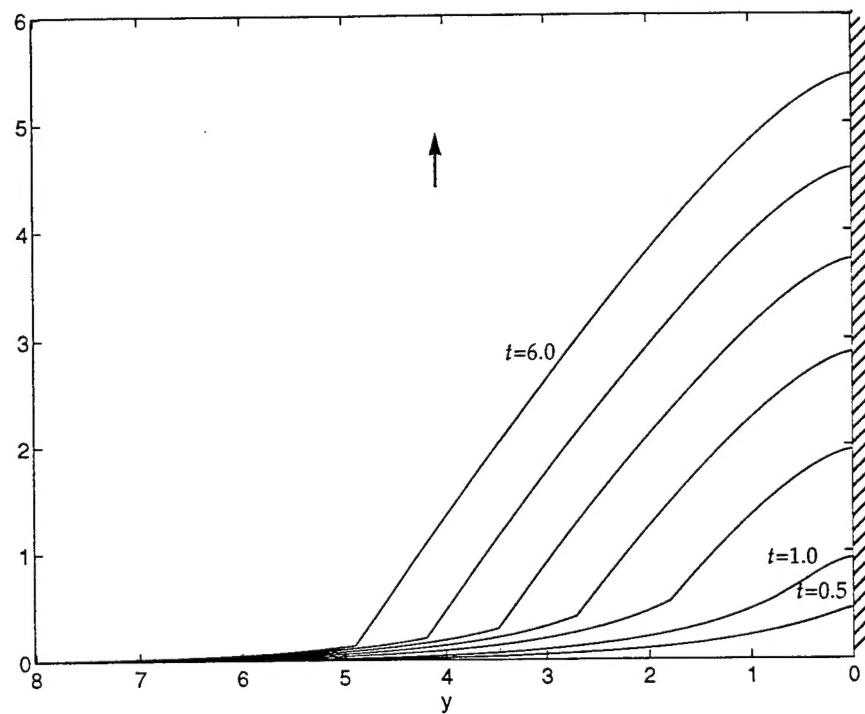


Fig.3 The solution of the initial value problem showing the shape of the jump at different times. Initially the jump is a straight line normal to the coast. View from above. The distances are scaled by the Rossby radius.

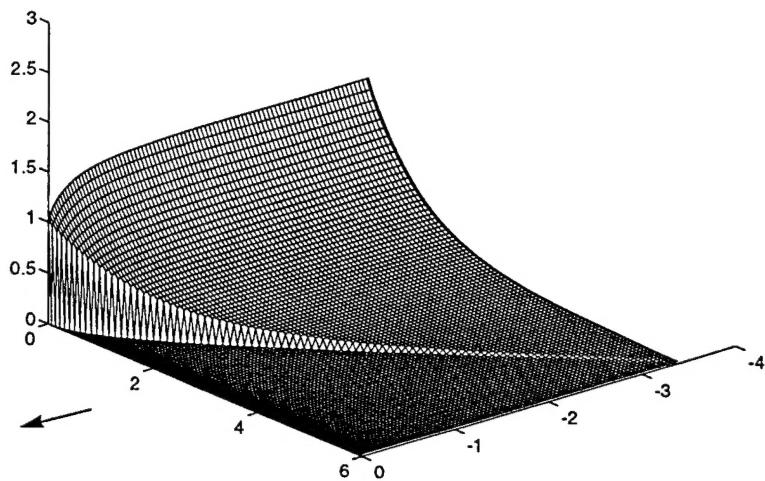


Fig.4 A developed Kelvin jump propagating along the coast. The graph displays the isopycnal displacement. The direction of propagation is shown by the arrow.

some distance offshore the jump becomes oblique to the coastline, and the final shape of the jump and its speed depend only on the jump strength. The shape of the resulting jump solution, especially for larger amplitudes, has a strong resemblance to the satellite imagery of overcast propagating to the north along the western coast of North America in the atmospheric marine layer (Mass and Albright 1987).

The Kelvin jumps give rise to a moderate offshore flow. The net offshore flow is nonzero. In general, both freely-propagating nonlinear Kelvin waves and Kelvin jump can be actively engaged in across-shelf mixing in coastal oceans. They can also easily transfer the energy of longer scale motion to shorter scales, leading to the formation of turbulence and energy dissipation. Kelvin jumps especially provide a good model for the frontogenesis and frontal propagation in coastal oceans and the atmosphere.

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